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The quasi-elastic nuclear response

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We explore the nuclear responses at intermediate energies, particularly in the spin longitudinal $\sigma \cdot q$ and spin transverse $\sigma \times q$ isovector channels, within the continuum random phase approximation framework. We also employ an extension of the standard random phase approximation to account for the spreading width of the single particle states through the inclusion of a complex and energy-dependent nucleon self-energy. The nuclear responses are then used as the basic ingredient to calculate hadronic reactions in the Glauber theory framework. Here both one and two-step contributions to the multiple scattering series in the quasi-elastic peak region are taken into account. We find evidence for shell effects in the one-step response and a strong dependence on the momentum regime of the two-step contribution.

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I. INTRODUCTION

The interest in the nuclear responses at intermediate energies, in particular in the quasi-elastic peak (QEP) region, is still widespread: indeed, the body of experimental data presently available in this energy regime is not easily amenable to a unified and consistent understanding.

For example, the maxima of the (p,n) cross-sections [1] appear hardened, over a wide range of momentum transfers, by about 25 MeV with respect to the ones observed in the (p,p′) reaction [2], an outcome still waiting for a convincing interpretation in terms of either many-body or reaction-mechanism effects.

The issue is further complicated by the experiments with heavier probes: in fact, the hardening of the QEP position found in the (^{3}He,t) [3] and in the (d,2p) [4] reactions at low transferred momenta turns into a softening when the momentum transfer is increased. Although this behaviour might be related to the composite nature of the probes, it is notable that the same effect it is observed also in pion single charge-exchange reactions [5]. On the other hand, the predicted hardening of the isovector transverse $\boldsymbol{\sigma}\times\mathbf{q}$ response appears well established according to the available data of deep inelastic (e,e′) scattering [6].

In this paper, we shall mainly concentrate on the $\boldsymbol{\sigma}\cdot\mathbf{q}$ and $\boldsymbol{\sigma}\times\mathbf{q}$ isovector channels. Actually, some evaluations of the spin-isospin ($\sigma\tau$) responses have already been performed in the framework of the bound state [7–11] or continuum [12,13] random phase approximation (RPA) with the conventional particle-hole (ph) interaction for the $\sigma\tau$ channel. As it is well known, the latter includes, beside the Landau-Migdal parameter g' (or, equivalently, a short range interaction of some sort), the exchange of the pion and the rho meson.

A suitable test for these calculations is represented by the above mentioned data of deep inelastic electron scattering, where the $\boldsymbol{\sigma}\times\mathbf{q}$ response has been separated out in a few nuclei over a wide range of momentum transfers. Here the virtual photon is exploring the whole of the nucleus, which thus has all its constituents responding to the probe.

When, however, one comes to consider strongly interacting probes, the ones offering a real hope of disentangling the elusive $\boldsymbol{\sigma}\cdot\mathbf{q}$ response, the additional problem is faced of appropriately dealing with the reaction mechanism, or, in other words, with the distortion of the impinging and outgoing hadronic waves. This is essential, since here the response of the nucleus is mostly confined to the surface region and one has to accurately assess how much of the latter is actually involved in the process.

In this connection, the authors of Ref. [13] resorted to the Distorted Wave Born Approximation (DWBA), whereas in Refs. [10,11] the Glauber theory [14] in the one-step approximation was employed. The former approach, as it is well-known, is fully quantum-mechanical; however, in the high energy limit and for nearly forward scattering the DWBA practically coincides with the eikonal approximation and thus is equivalent to the Glauber theory: but the latter allows one to deal with multistep processes, which are likely to give a substantial contribution at large momentum and energy transfers.

This is indeed what happens and we shall explore the contribution the two-step processes provide, particularly in charge-exchange reactions, where their importance is expected to be greater.

Turning to the many-body aspect of the problem, we calculate the $\sigma\tau$ nuclear responses in the continuum RPA with the $(g'+\pi+\rho)$ ph interaction, but utilizing a method different from

the one originally proposed by Shlomo and Bertsch [15], which has been widely exploited in this kind of calculations. Our approach is entirely worked out in momentum rather than in coordinate space and allows one to incorporate in a natural way the spreading width of the ph states.

In fact, continuum RPA naturally accounts for the escape width of the particle states, ignored, instead, in a harmonic oscillator basis. However, the spreading width of the states is also quite important for a realistic description of the $\sigma\tau$ responses. The inclusion of the latter is achieved in Ref. [13] through the direct introduction of an optical potential in the Schroedinger equation for the single particle (but not for the hole) states. The disadvantage of such an approach lies in the ensuing lack of orthogonality between the single particle wave functions that needs then to be cured with a rather cumbersome procedure [16].

Alternatively, Smith and Wambach [17] introduced a phenomenological complex and energy-dependent self-energy that directly couples the ph to the 2p2h sector of the Hilbert space, a procedure that should simulate the much more computer-time consuming Second RPA (SRPA) [18]. We adopt here an analogous approach: however, instead of introducing the coupling at the level of the RPA eigenstates we do so at the level of the bare ph states and *then* proceed to the construction of the RPA solution. The two approaches differ since the validity of the former requires the near diagonality and state independence of the "collective" self-energy in the RPA basis, whereas in the latter the ph self-energy in the ph basis is required to be almost diagonal. Notably, the two frameworks lead to quite similar results, as far as the nuclear response is concerned.

In this paper we shall test the above outlined model against deep inelastic electron scattering data (*volume responses*) on ^{40}Ca as obtained in Saclay [6]. For the *surface responses* we shall analyze the data obtained with the charge exchange (p,n) reaction at Los Alamos [1]. We shall consider as well the (p,p$'$) data [2]. In both cases the proton probe has an energy of 795 MeV. This set of data represents a large body of what is nowadays available on the experimental side. We do not analyze here the (d,2p) and (^{3}He,t) data (see, however, Ref. [11]): the internal structure of composite probes would require an appropriate treatment, since it has a strong impact on how the probe's and target's spins are coupled [19], and might also qualitatively influence the distortion effects [20].

We organize the paper as follows: in Sec. II we present our formalism of the continuum RPA and the way we deal with the spreading width problem; in Sec. III we discuss the one and two-step contributions to the surface response within the Glauber theory, taking into proper account the cylindrical geometry of the reaction.

In Sec. IV we show how the various ingredients of the model affect the response functions and then test the model against the already mentioned body of experimental data.

Finally, in the concluding Section we summarize our results.

II. VOLUME RESPONSE FUNCTIONS

A. The Polarization Propagator and the Continuum RPA

As it is well-known, the nuclear response function to an external probe is obtained through the imaginary part of the polarization propagator [21] (non-diagonal in momentum space for a finite system), which reads

$$
\Pi(\mathbf{q},\mathbf{q}';\omega) = \sum_{n\neq 0} <\psi_0|\hat{O}(\mathbf{q})|\psi_n><\psi_n|\hat{O}^\dagger(\mathbf{q}')|\psi_0>
\times\left[\frac{1}{\hbar\omega-(E_n-E_0)+i\eta}-\frac{1}{\hbar\omega+(E_n-E_0)-i\eta}\right], \tag{2.1}
$$

where

$$
\hat{H}|\psi_n>= E_n|\psi_n>, \tag{2.2}
$$

$\hat{H}$ being the full nuclear Hamiltonian and $\hat{O}(\mathbf{q})$ the second quantized expression of the vertex operator. We shall mainly be concerned with the $\sigma\tau$ nuclear excitations: in such a case one has

$$
O_L(\mathbf{q},\mathbf{r}) = \tau_\alpha \boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}} \tag{2.3a}
$$

in the *longitudinal* channel and

$$
O_T(\mathbf{q},\mathbf{r}) = \frac{\tau_\alpha}{\sqrt{2}}\boldsymbol{\sigma}\times\hat{\mathbf{q}}\mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}} \tag{2.3b}
$$

in the *transverse* one. In the following, for sake of convenience, we shall set $E_0=0$.

The angular part of $\Pi(\mathbf{q},\mathbf{q}';\omega)$ can be handled through a multipole decomposition which reads [22]

$$
\Pi_L(\mathbf{q},\mathbf{q}';\omega) = \sum_{JM}\Pi_J(q,q';\omega)Y^*_{JM}(\hat{\mathbf{q}})Y_{JM}(\hat{\mathbf{q}}') \tag{2.4}
$$

in the $\boldsymbol{\sigma}\cdot\mathbf{q}$ channel and

$$
\Pi_T(\mathbf{q},\mathbf{q}';\omega) = \sum_{JJ'M'}\Pi_{JJ'}(q,q';\omega)Y^*_{J'M'}(\hat{\mathbf{q}})Y_{J'M'}(\hat{\mathbf{q}}')\frac{2J+1}{2J'+1} \tag{2.5}
$$

in the $\boldsymbol{\sigma}\times\mathbf{q}$ one.

The latter expression, somewhat more involved than the former, is shortly derived in Appendix A. Also

$$
\Pi_J(q,q';\omega) = \sum_{\ell\ell'}\left[\hat{\Pi}_J(q,q';\omega)\right]_{\ell\ell'} a_{J\ell}a_{J\ell'} \tag{2.6a}
$$

and

$$
\Pi_{JJ'}(q,q';\omega) = \frac{1}{2}\sum_{\ell\ell'}\left[\hat{\Pi}_J(q,q';\omega)\right]_{\ell\ell'} b^{J'}_{J\ell}b^{J'}_{J\ell'}, \tag{2.6b}
$$

where

$$
a_{J\ell} = (-1)^\ell\sqrt{2\ell+1}\begin{pmatrix}\ell & 1 & J\\ 0 & 0 & 0\end{pmatrix}. \tag{2.7a}
$$

and

$$b_{J\ell}^{J'} = \sqrt{6}\sqrt{2J'+1}\, a_{J'\ell} \left\{ \begin{array}{ccc} J & J' & 1 \\ 1 & 1 & \ell \end{array} \right\}. \tag{2.7b}$$

From (2.4) and (2.5) the following expressions for the $\sigma\tau$ nuclear responses are then obtained:

$$\begin{aligned} R_{L,T}(q,\omega) &= -\frac{1}{\pi}\mathrm{Im}\Pi_{L,T}(q,q,;\omega) \\ &= -\frac{1}{4\pi^2}\mathrm{Im}\sum_J (2J+1)\Pi_{J(L,T)}(q,q;\omega), \end{aligned} \tag{2.8}$$

with

$$\begin{aligned} \Pi_{J(L)}(q,q';\omega) &\equiv \Pi_J(q,q';\omega) \\ &= \sum_{\ell\ell'} \left[\hat{\Pi}_J(q,q';\omega)\right]_{\ell\ell'} a_{J\ell}a_{J\ell'} \end{aligned} \tag{2.9a}$$

and

$$\begin{aligned} \Pi_{J(T)}(q,q';\omega) &= \sum_{J'} \Pi_{JJ'}(q,q';\omega) \\ &= \frac{1}{2}\sum_{\ell\ell'} \left[\hat{\Pi}_J(q,q';\omega)\right]_{\ell\ell'} (\delta_{\ell\ell'} - a_{J\ell}a_{J\ell'}). \end{aligned} \tag{2.9b}$$

The evaluation of the J-th multipole $\hat{\Pi}_J$, the *dynamical part* of the polarization propagator, is carried out in this paper in the RPA framework by solving the following set of coupled (by the tensor interaction) integral equations [22]

$$\begin{aligned} \left[\hat{\Pi}_J^{\mathrm{RPA}}(q,q';\omega)\right]_{\ell\ell'} &= \left[\hat{\Pi}_J^0(q,q';\omega)\right]_{\ell\ell'} \\ &+ \frac{1}{(2\pi)^3}\int_0^\infty dk\, k^2 \sum_{\ell_1\ell_2} \left[\hat{\Pi}_J^0(q,k;\omega)\right]_{\ell\ell_1} \left[U_J(k,\omega)\right]_{\ell_1\ell_2} \left[\hat{\Pi}_J^{\mathrm{RPA}}(k,q';\omega)\right]_{\ell_2\ell'}. \end{aligned} \tag{2.10}$$

The bare ph polarization propagator (obtained by replacing the full nuclear hamiltonian $\hat{H}$ with the mean field one $\hat{H}^0$) reads

$$\left[\hat{\Pi}_J^0(q,q';\omega)\right]_{\ell\ell'} = \sum_{ph} Q_{ph}^{J\ell}(q) \left[\frac{1}{\hbar\omega - (\epsilon_p - \epsilon_h) + i\eta} - \frac{1}{\hbar\omega + (\epsilon_p - \epsilon_h) - i\eta}\right] Q_{ph}^{J\ell'*}(q'), \tag{2.11a}$$

where

$$\begin{aligned} Q_{ph}^{J\ell}(q) &= <j_p j_h; J|\ell\sigma; J> \delta_{\sigma,1}(-i)^{\ell+1}(-1)^{\ell_h} 4[\pi(2\ell_p+1)(2\ell_h+1)]^{1/2} \\ &\times \mathcal{I}_{\ell n_p \ell_p j_p n_h \ell_h j_h}(q) \begin{pmatrix} \ell_p & \ell_h & \ell \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{2.11b}$$

In Eq. (2.11b) $<j_p j_h;J|\ell\sigma;J>$ is the standard $LS-jj$ recoupling coefficient and[1]

$$\mathcal{I}_{\ell n_p\ell_p j_p n_h\ell_h j_h}(q) = \int_0^\infty dr\, r^2 j_\ell(qr) R_{n_p\ell_p j_p}(r) R_{n_h\ell_h j_h}(r). \tag{2.11c}$$

If the spin-orbit term of the nuclear mean field is neglected, then (2.11a) is diagonal in the orbital angular momentum, i. e. $[\hat{\Pi}^0_J(q,q';\omega)]_{\ell\ell'} = \delta_{\ell\ell'}\hat{\Pi}^0_\ell(q,q';\omega)$, where

$$\begin{aligned}\hat{\Pi}^0_\ell(q,q';\omega) = 16\pi \sum_{\substack{n_p\ell_p\\ n_h\ell_h}} (2\ell_p+1)(2\ell_h+1) \begin{pmatrix} \ell_p & \ell_h & \ell \\ 0 & 0 & 0 \end{pmatrix}^2 \mathcal{I}_{\ell n_p\ell_p n_h\ell_h}(q)\, \mathcal{I}_{\ell n_p\ell_p n_h\ell_h}(q') \\ \times \left[\frac{1}{\hbar\omega - (\epsilon_{n_p\ell_p} - \epsilon_{n_h\ell_h}) + i\eta} - \frac{1}{\hbar\omega + (\epsilon_{n_p\ell_p} - \epsilon_{n_h\ell_h}) - i\eta} \right].\end{aligned} \tag{2.12}$$

In (2.11a), (2.11b) and (2.11c) $R_{p(h)}$ are the radial wave functions and $\epsilon_{p(h)}$ the associated eigenvalues. They are obtained by solving the Schroedinger equation with the Woods-Saxon potential

$$W(r) = \frac{W_0}{1+\mathrm{e}^{(r-R)/a}} + \left[\frac{\hbar c}{m_\pi^2 c^2}\right]^2 \frac{W_{so}}{ar} \frac{\mathrm{e}^{(r-R)/a}}{[1+\mathrm{e}^{(r-R)/a}]^2}\, \boldsymbol{\ell}\cdot\boldsymbol{\sigma} \tag{2.13}$$

(in the present paper, for sake of simplicity, the Coulomb potential is neglected), where m_π is the pion mass. Obviously, for the states in the continuum part of the spectrum the sum over n_p has to be changed into an integral, since the principal quantum number n_p becomes a continuum variable. Instead of following the standard procedure of calculating the propagator in the coordinate space and then Fourier transform to the momentum space, we have chosen to evaluate directly Eq. (2.11a): the calculation is fast and straightforward and one only needs to take some care in performing the integrals over the ph energies in the resonance region.

As already mentioned, in the RPA equations we employ the ph interaction $g' + V_\pi + V_\rho$, namely

$$[U_J(k,\omega)]_{\ell_1\ell_2} = V_L(k,\omega) a_{J\ell_1} a_{J\ell_2} + V_T(k,\omega)(\delta_{\ell_1\ell_2} - a_{J\ell_1} a_{J\ell_2}), \tag{2.14}$$

where

$$\begin{aligned} V_L(k,\omega) &= \Gamma_\pi^2(k_\mu^2)\frac{f_\pi^2}{\mu_\pi^2}\left[g' + \frac{k^2}{\omega^2 - k^2 - \mu_\pi^2}\right] \\ V_T(k,\omega) &= \left[\Gamma_\pi^2(k_\mu^2)\frac{f_\pi^2}{\mu_\pi^2} g' + \Gamma_\rho^2(k_\mu^2)\frac{f_\rho^2}{\mu_\rho^2}\frac{k^2}{\omega^2 - k^2 - \mu_\rho^2}\right]. \end{aligned} \tag{2.15}$$

---

[1]With respect to previous work [22], we have changed here the definition of $\hat{O}_{L,T}$ in (2.3a) and (2.3b) by using the unit versor $\hat{\mathbf{q}}$ instead of the vector $\mathbf{q}$ in the spin longitudinal and transverse operators. Accordingly, the quantity $\mathcal{I}$ is now defined without a factor q in front whereas in the ph interaction (2.15) appears an additional factor k^2. Also, a factor $1/\sqrt{2}$ has been introduced in the (2.3b) in order to have the same normalization for both the longitudinal and transverse propagators.

In the above, $f_\pi^2/4\pi\hbar c = 0.08$, $f_\rho^2/\mu_\rho^2 = 2.18(f_\pi^2/\mu_\pi^2)$, $k_\mu \equiv (\omega, \mathbf{k})$ and the usual monopole form factors

$$\Gamma_{\pi,\rho}(k_\mu^2) = \frac{\Lambda_{\pi,\rho}^2 - m_{\pi,\rho}^2}{\Lambda_{\pi,\rho}^2 - \omega^2 + k^2} \tag{2.16}$$

have been included at the $\pi(\rho)$NN vertices with cut-offs $\Lambda_\pi = 1300$ MeV and $\Lambda_\rho = 1700$ MeV, respectively. Moreover, in the following we shall always use $g' = 0.7$.

Since we shall also be interested in (p,p′) reactions, we have to introduce, in addition to the spin-isospin responses, a response function R_{ST} in the scalar ($S = 0$) and isoscalar ($T = 0$) channel ((p,p′) scattering at intermediate energies is in fact largely dominated by this channel). This response reads

$$R_{00}(q,\omega) = -\tfrac{1}{4\pi^2}\mathrm{Im}\textstyle\sum_J(2J+1)\Pi_{J(00)}(q,q;\omega), \tag{2.17}$$

where at zero-order $\Pi_{J(00)}(q,q';\omega)$ coincides with the $[\hat{\Pi}_J^0(q,q';\omega)]_{JJ}$ of Eq. (2.11a), substituting $\delta_{\sigma,1}$ with $\delta_{\sigma,0}$ in (2.11b). In RPA also $\Pi_{J(00)}$ obeys an integral equation similar to (2.10) and for the ph interaction we have used the G-matrix of Ref. [23] at a density of roughly one-half the central nuclear density.

B. The Spreading Width

The continuum RPA framework naturally accounts for the *escape width* of the particle states. However, also the *spreading width* of the latter plays an important role in the nuclear many-body problem and should be reckoned with. For this purpose, it is convenient, following Ref. [17], to recast the expression (2.1) for the polarization propagator as follows

$$\Pi(\mathbf{q},\mathbf{q}';\omega) =< \psi_0|\hat{O}(\mathbf{q})\, G(\omega)\, \hat{O}^\dagger(\mathbf{q}')|\psi_0 >, \tag{2.18}$$

where

$$G(\omega) = \frac{1}{\hbar\omega - \hat{H} + i\eta} - \frac{1}{\hbar\omega + \hat{H} - i\eta} \tag{2.19}$$

is the propagator of the excitation induced on the exact nuclear ground state $|\psi_0 >$ by the operator $\hat{O}(\mathbf{q})$.

Now, in the framework of Feshbach's formalism, starting from

$$\hat{H} = \hat{H}_0 + \hat{V} \tag{2.20}$$

one can derive the following effective Hamiltonian, restricted to operate in the ph space:

$$\begin{aligned}\hat{H}_{\mathit{eff}} &= P\hat{H}P - P\hat{V}Q\frac{1}{Q\hat{H}Q - \hbar\omega - i\eta}Q\hat{V}P \\ &= P\hat{H}P - U(\omega) \\ &= \hat{H}^0_{\mathit{eff}} + P\hat{V}P,\end{aligned} \tag{2.21}$$

having set

$$\hat{H}^0_{\mathit{eff}} = P\hat{H}_0 P - U(\omega). \tag{2.22}$$

In the above P and Q are the projection operators in the ph and in the complement space, respectively.

Therefore, to allow the formalism to encompass the coupling of the ph excitations to the 2p2h ones (or to more complicated configurations), i. e. to incorporate the spreading width of the ph states, one should replace in the previously outlined RPA scheme the "bare" ph propagator with the expression

$$\begin{aligned}\Pi^0(\mathbf{q},\mathbf{q}';\omega) = \sum_{ph} &< \phi_0|\hat{O}(\mathbf{q})|\phi_{ph} >< \phi_{ph}|\hat{O}^\dagger(\mathbf{q}')|\phi_0 > \\ &\times \left[\frac{1}{\hbar\omega + \Sigma_{ph}(\omega) - (\epsilon_p - \epsilon_h) + i\eta} - \frac{1}{\hbar\omega - \Sigma_{ph}(-\omega) + (\epsilon_p - \epsilon_h) - i\eta}\right],\end{aligned} \tag{2.23}$$

where

$$\Sigma_{ph}(\omega) =< \phi_{ph}|\hat{V}Q(Q\hat{H}Q - \hbar\omega - i\eta)^{-1}Q\hat{V}|\phi_{ph} > \tag{2.24}$$

and

$$\hat{H}_0|\phi_{ph} >= (\epsilon_p - \epsilon_h)|\phi_{ph} >, \tag{2.25}$$

the $|\phi_{ph} >$ obviously being the "bare" ph states.

A microscopic calculation of $\Sigma_{ph}(\omega)$ is difficult to carry out. An alternative approach [17], to which we shall adhere, assumes that the real and the imaginary parts of Σ_{ph}, namely

$$\Sigma_{ph}(\omega) = \Delta_{ph}(\omega) + i\frac{\Gamma_{ph}(\omega)}{2}, \tag{2.26}$$

can be cast into the form

$$\begin{aligned}\Gamma_{ph}(\omega) &= \gamma_p(\hbar\omega + \epsilon_h) + \gamma_h(\epsilon_p - \hbar\omega) \\ \Delta_{ph}(\omega) &= \Delta_p(\hbar\omega + \epsilon_h) + \Delta_h(\epsilon_p - \hbar\omega),\end{aligned} \tag{2.27}$$

where the arguments of the functions appearing in the right hand side of the above expressions are inferred from the analysis of the second order diagrams of Fig. 1(a). Note that the diagrams corresponding to the exchange of a bubble between two fermionic lines, conjectured to be small in the $\sigma\tau$ channel, are neglected (Fig. 1(b)).

Now, instead of calculating these diagrams, we utilize the formulae

$$\begin{aligned}\gamma_p(\epsilon) &= 2\alpha\left(\frac{\epsilon^2}{\epsilon^2 + \epsilon_0^2}\right)\left(\frac{\epsilon_1^2}{\epsilon^2 + \epsilon_1^2}\right)\theta(\epsilon) \\ \gamma_h(\epsilon) &= \gamma_p(-\epsilon),\end{aligned} \tag{2.28}$$

symmetrical with respect to the Fermi energy ($\epsilon_F = 0$), which give a reasonable fit of the particle widths for medium-heavy nuclei, using $\alpha = 10.75$ MeV, $\epsilon_0 = 18$ MeV and $\epsilon_1 = 110$ MeV [24]. We then insert these expressions into the once-subtracted dispersion relation

$$\Delta_p(\epsilon) = \frac{\epsilon}{2\pi}\,\mathrm{P}\int_0^\infty d\epsilon'\,\frac{\gamma_p(\epsilon')}{(\epsilon'-\epsilon)\epsilon'}\,, \tag{2.29}$$

(and in a similar one for Δ_h) obtaining

$$\begin{aligned}\Delta_p(\epsilon) &= \frac{1}{\pi}\frac{\alpha\epsilon_1^2}{\epsilon_1^2-\epsilon_0^2}\left\{\frac{\epsilon}{2}\left[\frac{\epsilon_0}{\epsilon^2+\epsilon_0^2}-\frac{\epsilon_1}{\epsilon^2+\epsilon_1^2}\right]-\epsilon^2\left[\frac{\log|\epsilon/\epsilon_0|}{\epsilon^2+\epsilon_0^2}-\frac{\log|\epsilon/\epsilon_1|}{\epsilon^2+\epsilon_1^2}\right]\right\}\\ \Delta_h(\epsilon) &= -\Delta_p(-\epsilon).\end{aligned} \tag{2.30}$$

The subtraction at the Fermi surface avoids the double counting of the smooth background in the single particle energy already embodied in the mean field.

Formulae (2.28) and (2.30) are the ones we shall employ in analyzing the $\sigma\tau$ nuclear response functions, dressing the bare ph propagator and then solving the RPA equations.

III. SURFACE RESPONSE FUNCTIONS

A. The Response Function in the One-Step Glauber Theory

In order to treat the response of the nucleus to a strongly interacting probe, one should enlarge the framework described in the previous Section to account for both the distortion and the absorption of the impinging probe on the surface of the nucleus.

In the framework of Glauber theory [14], the scattering amplitude of a probe on a nucleus of mass number A is given by

$$F_{fi}(\mathbf{q}) = \frac{ik}{2\pi}\int d\mathbf{b}\,e^{i\mathbf{q}\cdot\mathbf{b}} < \psi_f|\Gamma(\mathbf{b};\mathbf{s}_1...\mathbf{s}_A)|\psi_i >, \tag{3.1}$$

where the so-called nuclear profile function Γ is expressed as

$$\Gamma(\mathbf{b};\mathbf{s}_1...\mathbf{s}_A) = 1 - \prod_{j=1}^{A}[1-\Gamma_j(\mathbf{b}-\mathbf{s}_j)] \tag{3.2}$$

in terms of the single nucleon profile function

$$\Gamma_j(\mathbf{b}) = \frac{1}{2\pi ik}\int d\boldsymbol{\lambda}\,e^{-i\boldsymbol{\lambda}\cdot\mathbf{b}}f_j(\boldsymbol{\lambda}). \tag{3.3}$$

In the above, $\mathbf{b}$ is the impact parameter and $\mathbf{q} = \mathbf{k}-\mathbf{k}'$ the transferred momentum ($\mathbf{k}$ and $\mathbf{k}'$ are the projectile incoming and outgoing momenta, respectively): they are bi-dimensional vectors in the plane orthogonal to the direction of motion of the projectile. The probe-nucleon amplitudes $f(\boldsymbol{\lambda})$ of Eq. (3.3) are assumed to be the free ones and are meant to be evaluated in the laboratory system.

If the excitation energy of the nucleus is supplied by the probe in a single collision (one-step assumption), then, for large enough nuclei, one can rewrite Eq. (3.1) in the form

$$F_{fi}(\mathbf{q}) = \frac{ik}{2\pi} \int d\mathbf{b}\, e^{i\mathbf{q}\cdot\mathbf{b}} e^{i\chi_{\rm opt}(b)} < \psi_f | \Gamma_{\sigma\tau}(\mathbf{b}-\mathbf{s}) | \psi_i >, \tag{3.4}$$

where, for definiteness, we consider the spin-isospin inelastic channel. The complex phase shift $\chi_{\rm opt}$, responsible for the absorption and distortion of the probe, is given by

$$\chi_{\rm opt}(b) = \frac{2\pi}{k} f(0)\, T(b), \tag{3.5}$$

with

$$T(b) = \int_{-\infty}^{+\infty} dz\, \rho(r = \sqrt{b^2 + z^2}), \tag{3.6}$$

$\rho(r)$ being the nuclear density (assumed in the following to be represented by a Fermi distribution) and $f(0)$ the forward total NN scattering amplitude. At high energies, the imaginary part of $f(0)$ is dominant and, therefore, making use of the optical theorem, one can write

$$\chi_{\rm opt}(b) = \frac{i}{2} \tilde{\sigma}_{\rm tot}\, T(b), \tag{3.7}$$

where $\tilde{\sigma}_{\rm tot}$ is the effective probe-nucleon total cross-section (effective because empirically embodying Pauli blocking effects).

Eq. (3.4) suggests to replace the vertex operators $O_{L,T}(\mathbf{q}, \mathbf{r})$ of (2.3a) and (2.3b) with the new ones

$$O^{surf}_{L,T}(\mathbf{q}, \mathbf{r}) = \frac{1}{(2\pi)^2 f_{L,T}(q)} \int d\mathbf{b}\, d\boldsymbol{\lambda}\, e^{i\chi_{\rm opt}(b)} e^{i(\mathbf{q}-\boldsymbol{\lambda})\cdot\mathbf{b}} f_{L,T}(\lambda) O_{L,T}(\boldsymbol{\lambda}, \mathbf{r}), \tag{3.8}$$

where the $f_{L,T}(\lambda)$ are the elementary isovector spin-longitudinal and spin-transverse probe nucleon scattering amplitudes. From the above expression one sees that the probe does not transfer a single momentum $\mathbf{q}$ to the nucleus, but rather all possible momenta $\boldsymbol{\lambda}$ with weight $f_{L,T}(\lambda)$. Furthermore, its distortion is controlled by the factor $\exp[i\chi_{\rm opt}(b)]$. The normalization in Eq. (3.8) has been chosen in order to recover the standard vertex operators in the limit of no distortion:

$$O^{surf}_{L,T} \stackrel{\tilde{\sigma}_{\rm tot}\to 0}{\longrightarrow} O_{L,T}. \tag{3.9}$$

For sake of simplicity, we have not explicitly expressed the dependence of $O^{surf}_{L,T}$ on the spin-isospin operators of the probe (see, however, Appendix B). If we insert the pertinent surface vertex operators in Eq. (2.1) and take the appropriate matrix elements of the spin and isospin probe operators, we can finally define two new polarization propagators $\Pi^{surf}_{L,T}(\mathbf{q}, \mathbf{q}'; \omega)$. The associated one-step response functions, $R^{(1)surf}_{L,T}(q, \omega)$, can then be obtained by substituting, in Eq. (2.8), $\Pi_{J(L,T)}(q, q; \omega)$ with the corresponding surface expressions, $\Pi^{surf}_{J(L,T)}(q, q; \omega)$, given by

$$
\begin{aligned}
\Pi^{surf}_{J(L)}(q,q;\omega) = {} & \Pi_{J(L)}(q,q;\omega) \\
& + \frac{1}{|f_L(q)|^2}\int_0^\infty d\lambda\,\lambda \int_0^\infty d\lambda'\,\lambda'\,\mathrm{Re}[f_L^*(\lambda)\,f_L(\lambda')\,G_J(\lambda,\lambda';q)]\Pi_{J(L)}(\lambda,\lambda';\omega) \\
& -2\frac{1}{|f_L(q)|^2}\int_0^\infty d\lambda\,\lambda\,\mathrm{Re}[f_L^*(q)\,f_L(\lambda)\,H_J(\lambda;q)]\Pi_{J(L)}(q,\lambda;\omega)
\end{aligned}
\tag{3.10a}
$$

and

$$
\begin{aligned}
\Pi^{surf}_{J(T)}(q,q;\omega) = {} & \Pi_{J(T)}(q,q;\omega) \\
& + \frac{1}{|f_T(q)|^2}\int_0^\infty d\lambda\,\lambda \int_0^\infty d\lambda'\,\lambda' \sum_{J'} \mathrm{Re}[f_T^*(\lambda)\,f_T(\lambda')\,G_{J'}(\lambda,\lambda';q)]\Pi_{JJ'}(\lambda,\lambda';\omega) \\
& -2\frac{1}{|f_T(q)|^2}\int_0^\infty d\lambda\,\lambda \sum_{J'} \mathrm{Re}[f_T^*(q)\,f_T(\lambda)\,H_{J'}(\lambda;q)]\Pi_{JJ'}(q,\lambda;\omega).
\end{aligned}
\tag{3.10b}
$$

In Eqs. (3.10a) and (3.10b) we have set

$$
\begin{aligned}
G_J(\lambda,\lambda';q) &= \sum_{\ell m} c_{J\ell m}\, g_m^*(\lambda,q) g_m(\lambda',q) \\
H_J(\lambda;q) &= \sum_{\ell m} c_{J\ell m}\, g_m(\lambda,q),
\end{aligned}
\tag{3.11}
$$

where

$$
g_m(\lambda,q) = \int_0^\infty db\, b\,\{1-\exp[i\chi_{\rm opt}(b)]\}\, J_m(\lambda b)\, J_m(qb) \tag{3.12}
$$

and

$$
\begin{aligned}
c_{J\ell m} &= I_{\ell+m} a^2_{J\ell} \frac{(\ell-m-1)!!(\ell+m-1)!!}{(\ell+m)!!(\ell-m)!!} \\
I_{\ell+m} &= \left\{ \begin{array}{ll} 0, & \ell+m \ \text{ odd} \\ 1, & \ell+m \ \text{ even} \end{array} \right. .
\end{aligned}
\tag{3.13}
$$

$\Pi_{JJ'}$ has been defined in Eq. (2.6b) or (2.9b). In Appendix B, we shortly sketch the derivation of Eqs. (3.10a) and (3.10b).

Again, also in the scalar-isoscalar channel, one can introduce a surface propagator $\Pi^{surf}_{J(00)}$: it is easily verified that its expression is identical to Eq. (3.10a), replacing everywhere (L) with (00) (f_{00} is the $S=0$, $T=0$ NN amplitude) and setting $a^2_{J\ell} \to \delta_{J,\ell}$ in (3.13).

B. The Response Function in the Two-Step Glauber Theory

Multiple scattering contributions to the nuclear response function can be calculated along lines similar to the one-step case. However, the problem gets numerically rather involved and therefore we resort to an approximation that is often employed in Glauber calculations.

In this approximation, each contribution from the multiple scattering series to the response function is expressed as a volume response function times a distortion factor [25,26]. The latter is independent of the transferred energy and momentum and should embody the distortion effects. Thus, for instance, the one-step response can be written as

$$R^{(1)}_{L,T}(q,\omega) = \mathcal{D}_1\, R_{L,T}(q,\omega), \tag{3.14}$$

where $R_{L,T}(q,\omega)$ is the volume response of Eq. (2.8) and

$$\mathcal{D}_1 = N^{(1)}_{\text{eff}} = \int d\mathbf{b}\, T(b) e^{-\tilde{\sigma}_{\text{tot}} T(b)}, \tag{3.15}$$

$N^{(1)}_{\text{eff}}$ being the effective number of nucleons participating in the single collision.

A word of caution is in order here: expression (3.14) is able to reproduce the gross features of the nuclear response function (say, its size), but yields the same shape of the volume response. This is at variance, as we shall see in the next Section (see also Ref. [11]), with the full Glauber calculation of the previous Subsection, since there the response is also reshaped, a feature that cannot be overlooked, if one has to disentangle genuine nuclear correlations from distortion effects. This may be also the case, of course, when one comes to multistep processes. Anyway, the size of their contribution, although not negligible, is considerably smaller than the one arising from the one-step scattering, thus reducing the sensitivity of the response to the details of their shape. This assumption has, however, to be assessed (see next Section).

For charge-exchange reactions at intermediate energies the two-step response function can then be defined as [26]

$$\begin{aligned} R^{(2)}_{L,T}(q,\omega) = \frac{\mathcal{D}_2}{k^2} \frac{2}{|f_{L,T}(q)|^2} \int d\mathbf{q}' \int_0^\omega d\omega' |f_{L,T}(q')|^2 R_{L,T}(q',\omega') \\ \times |f_{00}(|\mathbf{q}-\mathbf{q}'|)|^2 R_{00}(|\mathbf{q}-\mathbf{q}'|, \omega-\omega'), \end{aligned} \tag{3.16}$$

where $\mathbf{q}'$ is again a bi-dimensional vector, k is the momentum of the projectile and

$$\mathcal{D}_2 = \frac{1}{2} \int d\mathbf{b}\, T^2(b) e^{-\tilde{\sigma}_{\text{tot}} T(b)} \tag{3.17}$$

is connected to the effective number of pairs participating in the double scattering according to

$$N^{(2)}_{\text{eff}} = \binom{A}{2} \mathcal{D}_2(\tilde{\sigma}_{\text{tot}})/\mathcal{D}_2(0). \tag{3.18}$$

In Eq. (3.16), the charge-exchange reaction, driven by the spin-isospin amplitudes, can occur only once and the second scattering is driven by the scalar-isoscalar amplitudes. The factor 2 comes from the two possible orderings of the reaction.

Non-charge-exchange reactions, on the other hand, are dominated by the scalar-isoscalar channel, leading to the following definition:

$$\begin{aligned} R^{(2)}_{00}(q,\omega) = \frac{\mathcal{D}_2}{k^2} \frac{1}{|f_{00}(q)|^2} \int d\mathbf{q}' \int_0^\omega d\omega' |f_{00}(q')|^2 R_{00}(q',\omega') \\ \times |f_{00}(|\mathbf{q}-\mathbf{q}'|)|^2 R_{00}(|\mathbf{q}-\mathbf{q}'|, \omega-\omega'). \end{aligned} \tag{3.19}$$

Note that by inspecting (3.16) and (3.19) one can already predict that two-step contributions will be twice more important in charge-exchange reactions than in non-charge-exchange ones. Indeed, the fact that in $R^{(2)}_{L,T}$ one of the two rescatterings is driven by $|f_{00}|^2$ makes $R^{(2)}_{L,T}$ and $R^{(2)}_{00}$ practically of the same size, apart from the factor 2 due to the two orderings of the charge-exchange reaction.

IV. RESULTS

Let us start by discussing the *volume* responses, relevant to electron scattering. We have performed the calculations for ^{12}C and ^{40}Ca, using for the Woods-Saxon potential (2.13) the following set of parameters:

$$\begin{aligned} W_0 &= -54.8\,\text{MeV}, \quad W_{so} = -10\,\text{MeV}, \\ R &= 1.27\,A^{1/2}\,\text{fm}, \qquad a = 0.67\,\text{fm}. \end{aligned} \tag{4.1}$$

In Fig. 2 we display the $\sigma\tau$ longitudinal and transverse response functions of ^{40}Ca at two values of the momentum transfer q, comparing the free and RPA responses with and without inclusion of the spreading width of the ph states. When the spreading width is included, one observes a sizable damping of the QEP and of the low-energy resonances, together with a broadening of their width (since the energy sum rule has to be conserved). The effect is essentially the same in the free as in the RPA responses.

Note, as already mentioned in the Introduction, that in spite of the different approximations for the inclusion of the collisional damping of the single-particle motion underlying our approach and the one of Ref. [17], the two calculations are in fact in substantial agreement. Note also that these correlations, related to the single-particle motion, affect the position of the QEP much less than the RPA ones.

The transverse response function enters directly into the expression for the nuclear transverse structure function, measured in electron scattering experiments. Indeed, one has

$$S_T(q,\omega) = \frac{\mu_0^2}{e^2}(\mu_p - \mu_n)^2 G_M^2(q_\mu^2) R_T(q,\omega), \tag{4.2}$$

μ_0 being the nuclear Bohr magneton, $\mu_p = 2.79$, $\mu_n = -1.91$ and

$$G_M(q_\mu^2) = \frac{1}{[1 + (\mathbf{q}^2 - \omega^2/c^2)/18.1\,\text{fm}^{-2}]^2} \tag{4.3}$$

the usual electromagnetic γNN form factor. In (4.2) the small isoscalar contribution has been neglected.

In Fig. 3 we compare the calculations (with inclusion of the spreading width) of the previous figure to the experimental data [6]. It is quite clear that the RPA correlations are successful in bringing the QEP position to the right place, but they miss the correct q-dependence of the strength. It is unlikely that this shortcoming be due to the specific model we employ for the ph interaction: in our treatment the Landau-Migdal parameter g' incorporates the effect of the exchange diagrams in the RPA series and this approximation is known to work well in the spin-isospin channel [27].

Thus, to improve the accord with experiment, further contributions to the response function should likely be looked for beyond the ph Hilbert space of RPA, such as 2p-2h and meson exchange current terms. This is further suggested by the fact that in the high energy region of the response some strength is missing even at high momenta, where the strength of the QEP is fairly close to the data. See, for instance, Ref. [28] for a treatment of 2p-2h contributions in nuclear matter, where, indeed, a satisfactory description of (e,e′) data is found.

Let us now turn to the main topic of this work, namely to the reactions involving strongly interacting probes. In this case, a new feature of the response functions is related to the strong distortion of the projectile, whose strength is set by the effective total probe-nucleon cross-section of (3.7).

In Figs. 4 and 5 one can see a comparison of the one-step longitudinal and transverse *volume* (no distortion) and *surface* (distorted) responses for ^{12}C and ^{40}Ca at two transferred momenta, $q = 1.54$ fm^{-1} and $q = 2.31$ fm^{-1}. In each plot besides the volume response ($\widetilde{\sigma}_{\rm tot} = 0$), two surface cases ($\widetilde{\sigma}_{\rm tot} = 30$ mb and $\widetilde{\sigma}_{\rm tot} = 40$ mb) are displayed, for both the free and the RPA responses. As it may be expected, one observes a strong quenching of the surface responses, together with a reduction of the importance of the RPA correlations, since the reaction is now mainly confined at the low density, peripheral region of the nucleus.

Furthermore, another effect shows up, of great importance for the interpretation of the experimental data, namely a shift of the QEP position of the surface responses with respect to the volume ones: it appears to be sizable (due to the fact that RPA correlations are damped, it is much larger than the shift induced by the latter) and practically independent of $\widetilde{\sigma}_{\rm tot}$ in any realistic range of values for this parameter. It also turns out to be independent of the elementary NN amplitudes (here we employ the parameterization of Bugg and Wilkin [29]).

However, comparing the results for ^{12}C and ^{40}Ca, one also observes a shell dependence for this effect: this is better illustrated in Fig. 6, where the surface longitudinal response functions for ^{12}C and ^{40}Ca are directly compared for the case $\widetilde{\sigma}_{\rm tot} = 40$ mb at $q = 1.54$ fm^{-1} and $q = 2.31$ fm^{-1}, both with and without inclusion of the spreading width. In order to bring the responses at the same scale, R_L has been divided by $N_{\rm eff}$ (see (3.15)), with $N_{\rm eff}(^{12}{\rm C}) = 3.2$ and $N_{\rm eff}(^{12}{\rm Ca}) = 5.6$, respectively.

In Ref. [11] it has been shown that the ^{40}Ca one-step surface responses are hardened for $q \lesssim 1.8$ fm^{-1} and softened for $q \gtrsim 1.8$ fm^{-1}; in contrast, ^{12}C surface responses are always hardened, as one can see from Fig. 6. Furthermore, this feature is independent of the specific value of $\widetilde{\sigma}_{\rm tot}$, which is dictated by the energy and the nature of the projectile.

A contribution that might, in principle, reshape the quasi-elastic response functions is the two-step term in the Glauber multiple scattering expansion. In Ref. [26] it had been shown to produce at small transferred momenta a contribution smoothly increasing with the transferred energy. However, this feature, because of the NN amplitudes entering into (3.16) and (3.19), is strongly dependent upon the momentum regime.

To figure it out, we plot in Fig. 7 the two-step response $R^{(2)}$ for a simple model, namely assuming the two squared amplitudes in (3.19) to be equal and with a gaussian shape, $|f(q)|^2 = A\exp\left(-\eta q^2\right)$. Here, and also in the following calculations with realistic NN amplitudes, in order to estimate $R^{(2)}$ we use for simplicity the free harmonic oscillator model without inclusion of the spreading width: since $R^{(2)}$ is the convolution of two ph response

functions, it should be rather independent of the details of their shape.

The curves in Fig. 7 correspond to η ranging from 0.001 fm^2 to 1 fm^2 and it is quite apparent that the shape changes when the q-dependence of the amplitudes becomes more pronounced. The reason for this behaviour lies in (3.19): indeed, when $f(q')$ is a rapidly decreasing function, the main contribution to the integrals comes from the region $q' \sim 0$ and, as a consequence, $\omega' \sim 0$; it then follows that at fixed q the maximum of $R^{(2)}(q,\omega)$ will be found for ω around the QEP position. Note that for $\eta \gtrsim 0.1$ fm^2 one gets, at high momenta, a maximum *below* the QEP and that realistic values for η, which fit the NN amplitudes, fall just in this range.

Let us now see how our full model (namely one-step RPA Glauber responses with spreading width plus two-step free responses) compares to the available (p,n) data [1]. The double differential cross-section for a (p,n) reaction is given by

$$\frac{d^2\sigma}{d\Omega d\omega} = \sum_{\alpha=L,T} |f_\alpha(q)|^2 [R_\alpha^{(1)surf}(q,\omega) + R_\alpha^{(2)}(q,\omega)] \tag{4.4}$$

and it is tested in Fig. 8 on the data from a reaction on ^{12}C at a proton energy of 795 MeV and for four different scattering angles ($\theta = 9^o$, 12^o, 15^o and 18^o), corresponding to q ranging from 1.16 fm^{-1} to 2.31 fm^{-1}.

The dashed and the solid curves in each plot represent the one-step and the full calculations, respectively, whereas the dotted curve is the two-step term; for sake of comparison, we display also the cross-section corresponding to the simpler model based on the one-step contribution of Eq. (3.14) (dot-dashed line), i. e.

$$\frac{d^2\sigma}{d\Omega d\omega} = \sum_{\alpha=L,T} |f_\alpha(q)|^2 [R_\alpha^{(1)}(q,\omega) + R_\alpha^{(2)}(q,\omega)]. \tag{4.5}$$

A few comments are in order. First, the reduction in strength in going from (4.4) to (4.5) is due to RPA correlations (indeed, the difference fades away at large q), whereas the shift in the QEP position comes from the cylindrical geometry of the reaction, which is respected by our approach (it stays constant with q). Looking at the data, it is apparent that now the situation is complementary to what we have observed in the case of the (e,e') data: in fact, we are now able to reproduce the correct q-dependence of the strength, but not the QEP position of the (p,n) data, which is still somewhat more hardened than predicted by our calculations.

Note that the height of the peak depends on $\widetilde{\sigma}_{\rm tot}$: here, we have used $\widetilde{\sigma}_{\rm tot} = 40$ mb, without accounting for medium effects on this parameter. The estimate of this effective parameter is not well assessed, since there are large discrepancies between the direct calculation of Pauli blocking effects [25] and the derivation of $\widetilde{\sigma}_{\rm tot}$ from the optical potential [26]. Using $\widetilde{\sigma}_{\rm tot} = 30$ mb, the curves in Fig. 8 would get a 25% increasing at any momentum transfer, without affecting the q-dependence of the strength and reproducing correctly also the height of the peak.

The real problem with the model we are discussing is related to the A-dependence of the QEP position that it introduces (see Fig. 6), since the (p,n) data of Ref. [1] appear to scale, at least in ^{12}C and Pb. A scale factor is also the only difference in the (^{3}He,t) cross-sections on ^{12}C and ^{40}Ca [3], whose QEP position displays a pattern, as a function of

q, quite different from the (p,n) case, in agreement with our calculations for ^{40}Ca [11], but not for ^{12}C. In this connection, we remind the reader that in Ref. [11] the ^{3}He projectile has been treated as structureless, whereas, as already mentioned, a proper account of its internal structure might affect the shape of the response functions [20].

It is not clear, at the moment, whether this shell dependence of the Glauber quasi-elastic responses is a genuine effect or, rather, it reflects a shortcoming of the model or the approximations of our treatment: indeed, in the two-step contribution we have neglected the cylindrical geometry of the reaction, which, as we noted, is the source of the shift in the one-step term. Actually, the discrepancy between ^{12}C and ^{40}Ca shows up at large momenta (see Fig. 6), where the two-step response is relatively more important.

Finally, in Fig. 9 we show the same comparison as in Fig. 8, but for (p,p′) scattering at 795 MeV [2]. We have considered only the dominant scalar-isoscalar channel, which means that in (4.4) and (4.5) we have set $\alpha = 00$. By inspecting Fig. 9 a few observations follow: the two-step contribution is smaller than in the (p,n) case, as anticipated in Sec. III B; the simple calculation based on (3.14) of course yields a better account of the QEP position, since (p,p′) data do not show any shift; on the other hand, the q-dependence of the height of the peak is unsatisfactorily described, but this is due to our choice of only allowing the scalar-isoscalar channel: when the momentum transfer increases (say, over 1.5 fm^{-1}) other components of the NN amplitudes become important and should help in increasing the height of the response.

V. CONCLUSIONS

The aim of this paper has been to investigate the quasi-elastic nuclear response as seen in electron and, especially, proton inclusive scattering. We restricted ourselves to unpolarized scattering, since already in this domain a number of issues wait to be clarified.

Our attention has been mainly directed to the (p,n) and (p,p′) reactions, owing to the surprising and apparently contradictory figures displayed by these experiments. We have first applied our formalism, based on continuum RPA plus spreading width of the ph states, to the transverse electron scattering finding that:

i) RPA correlations shift the QEP position to higher energy;

ii) the q-dependence of the strength in the QEP domain is not correctly reproduced by our model, since some strength is actually missing at low but not at high momenta;

iii) in the high energy tail of the response strength is missing at all momenta.

The solution of the problem in items ii) and iii) likely comes from the contribution from 2p-2h processes and meson-exchange currents, as can be seen from the results of Ref. [28], worked out in nuclear matter. Neglecting 2p-2h contributions in hadron scattering should be a less serious shortcoming, because of the strong density dependence, which makes them relatively less important in the low density surface regions probed by strongly interacting projectiles.

In hadron scattering, on the other hand, multiple-scattering contributions can be substantial and, accordingly, we have adopted the framework of Glauber theory: one-step processes have been calculated consistently within the theory, whereas for two-step processes

we have resorted to the approximation of accounting for the effect of distortion through a multiplicative factor. From the analysis of the hadron responses we have established the following:

i) RPA correlations are strongly damped, but not completely washed out: this is at variance with the polarized ($\vec{\mathrm{p}}$,$\vec{\mathrm{n}}$) experiment in Los Alamos at $q \approx 1.72$ fm^{-1} [30], which seems to be compatible with free responses. It remains to be understood which mechanism is responsible of the complete suppression of the RPA correlations experimentally found;

ii) in contrast to (e,e$'$) scattering, the q-dependence of the strength is well reproduced (of course, properly accounting for the non-scalar-isoscalar channels in the (p,p$'$) reaction at high momenta), confirming the validity of the assumption that 2p-2h contributions should be negligible for hadronic reactions;

iii) the height of the QEP depends on the value of $\widetilde{\sigma}_{\mathrm{tot}}$, which is not well assessed: note that if one uses the free value, 40 mb, the height of the (p,p$'$) cross-section is well reproduced, whereas the height of the (p,n) one is underestimated;

iv) again in contrast to (e,e$'$) scattering, the QEP comes out in the wrong position: the calculations in ^{12}C predict a hardening of the cross-section in the whole range of momenta explored ($\sim 1 \div 3$ fm^{-1}), as it happens in the (p,n) but not in the (p,p$'$) reaction. Items iii) and iv) together apparently point to the need for contributions that appear only in (p,n) and not in (p,p$'$) scattering;

v) the shift of the peak in the calculated responses is a consequence of the distortion of the proton wave and it is *not* the same in ^{12}C and ^{40}Ca, whereas the available data appear to scale. Since the Glauber theory is equivalent to the distorted wave Born approximation in the eikonal limit (which might or might not be valid at the incident energy of 800 MeV), it would be interesting to see if also in the DWBA framework a shell dependence shows up. Note also that the two-step response is not treated in a way fully consistent with the Glauber theory, a fact that could have some influence on the shape of the response functions;

vi) at high transferred momenta the two-step term is sizable in the (p,n) reaction and, because of the momentum dependence of the NN amplitudes, does not give a smoothly increasing background, as it does at small momenta, but rather a contribution peaked slightly below the QEP.

No final statement can be made, at the moment, on these issues: however, we wish to stress the necessity of formulating a theory of the nuclear response able to cope simultaneously with all the different kinds of reactions. Accurate calculations are surely needed, but only from a careful cross-referencing of the many phenomena unveiled in the scattering on complex nuclei can one hope to obtain a guide towards a solution of the difficult many-body nuclear problem.

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APPENDIX A: THE TRANSVERSE POLARIZATION PROPAGATOR

The transverse polarization propagator is defined as the trace of the current-current propagator, namely

$$\Pi_T(\mathbf{q},\mathbf{q}';\omega) = \sum_n \Pi_{nn}(\mathbf{q},\mathbf{q}';\omega), \tag{A1}$$

where Π_{mn} is obtained by substituting $(\boldsymbol{\sigma}\times\hat{\mathbf{q}})_m$ and $(\boldsymbol{\sigma}\times\hat{\mathbf{q}})_n$ for the two vertex operators in Eq. (2.1) [22]; m and n are spherical indices.

Introducing vector spherical harmonics, Π_{mn} can be recast in the following form:

$$\Pi_{mn}(\mathbf{q},\mathbf{q}';\omega) = \sum_{\substack{JM \\ J_1 J_2}} \Pi_{J;J_1J_2}(q,q';\omega) Y^{(m)*}_{JJ_1M}(\hat{\mathbf{q}})\, Y^{(n)}_{JJ_2M}(\hat{\mathbf{q}}'), \tag{A2}$$

where

$$\Pi_{J;J_1J_2}(q,q';\omega) = \frac{1}{2}\sum_{\ell\ell'} \left[\hat{\Pi}_J(q,q';\omega)\right]_{\ell\ell'} b^{J_1}_{J\ell}\, b^{J_2}_{J\ell'}. \tag{A3}$$

The quantities in (A3) have been defined in Sec. II A. Using the addition theorem for the vector spherical harmonics, namely

$$4\pi \sum_{M=-J}^{J} \sum_m Y^{(m)*}_{JJ_1M}(\hat{\mathbf{q}})\, Y^{(m)}_{JJ_2M}(\hat{\mathbf{q}}') = \delta_{J_1J_2}(2J+1)P_{J_1}(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}'), \tag{A4}$$

one finds

$$\begin{aligned}\Pi_T(\mathbf{q},\mathbf{q}';\omega) &= \sum_{JJ'} \frac{(2J+1)}{4\pi} \Pi_{J;J'J'}(q,q';\omega) P_{J'}(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}') \\ &= \sum_{JJ'M'} \Pi_{J;J'J'}(q,q';\omega) Y^*_{J'M'}(\hat{\mathbf{q}}) Y_{J'M'}(\hat{\mathbf{q}}')\frac{2J+1}{2J'+1},\end{aligned} \tag{A5}$$

i. e., Eq. (2.5) with $\Pi_{JJ'}(q,q';\omega) = \Pi_{J;J'J'}(q,q';\omega)$.

Using Eq. (A3) the diagonal part of the transverse propagator can be written as

$$\Pi_T(\mathbf{q},\mathbf{q};\omega) = \sum_J \frac{2J+1}{4\pi} \sum_{\ell\ell'} \left[\hat{\Pi}_J(q,q';\omega)\right]_{\ell\ell'} \sum_{J'} b^{J'}_{J\ell}\, b^{J'}_{J\ell'}. \tag{A6}$$

It is easy to check that $\sum_{J'} b^{J'}_{J\ell} b^{J'}_{J\ell'} = \delta_{\ell\ell'} - a_{J\ell}a_{J\ell'}$ ($a_{J\ell}$ being defined in Eq. (2.7a)), from which the result of Eq. (2.9b) follows immediately.

APPENDIX B: THE SURFACE POLARIZATION PROPAGATOR

Here we briefly sketch the derivation of the longitudinal surface polarization propagator (3.10a). Similar calculations apply for the transverse one.

As mentioned in Sec. III A, the expression (3.8) for the surface vertex operator $O_L^{surf}(\mathbf{q},\mathbf{r})$ is oversimplified: actually, in (3.8) one should add, inside the momentum integral, the matrix element of the probe spin longitudinal operator, $< s_f|\boldsymbol{\sigma}^{(p)}\cdot\boldsymbol{\lambda}|s_i >$ (the treatment of the isospin is trivial). Inserting the resulting expression in the formula (2.1) for the polarization propagator, averaging over the initial probe spin s_i and summing over the final spin s_f one finds (for simplicity, we consider only the diagonal part of Π_L, i. e. $\mathbf{q}$=$\mathbf{q}'$)

$$\begin{aligned}
\Pi_L^{surf}(\mathbf{q},\mathbf{q};\omega) &= \frac{1}{(2\pi)^4}\frac{1}{|f_L(q)|^2}\int d\mathbf{b}\, d\boldsymbol{\lambda}\int d\mathbf{b}'\, d\boldsymbol{\lambda}'\, \hat{\boldsymbol{\lambda}}\cdot\hat{\boldsymbol{\lambda}}'\, f_L^*(\lambda)\mathrm{e}^{-i\chi^*_{\rm opt}(b)}\mathrm{e}^{-i(\mathbf{q}-\boldsymbol{\lambda})\cdot\mathbf{b}} \\
&\quad\times\Pi_L(\boldsymbol{\lambda},\boldsymbol{\lambda}';\omega)f_L(\lambda')\mathrm{e}^{i\chi_{\rm opt}(b')}\mathrm{e}^{i(\mathbf{q}-\boldsymbol{\lambda}')\cdot\mathbf{b}'} \\
&= \frac{1}{(2\pi)^4}\frac{1}{|f_L(q)|^2}\frac{4\pi}{3}\sum_{JM}\sum_{n}\int d\mathbf{b}\, d\boldsymbol{\lambda}\int d\mathbf{b}'\, d\boldsymbol{\lambda}' f_L^*(\lambda)\mathrm{e}^{-i\chi^*_{\rm opt}(b)}\mathrm{e}^{-i(\mathbf{q}-\boldsymbol{\lambda})\cdot\mathbf{b}} \\
&\quad\times\Pi_J(\lambda,\lambda';\omega)f_L(\lambda')\mathrm{e}^{i\chi_{\rm opt}(b')}\mathrm{e}^{i(\mathbf{q}-\boldsymbol{\lambda}')\cdot\mathbf{b}'}Y^*_{JM}(\hat{\boldsymbol{\lambda}})Y^*_{1n}(\hat{\boldsymbol{\lambda}})Y_{JM}(\hat{\boldsymbol{\lambda}}')Y_{1n}(\hat{\boldsymbol{\lambda}}'),
\end{aligned} \tag{B1}$$

where we have used the angular momentum expansion (2.4) and set $\hat{\lambda}_n = \sqrt{4\pi/3}Y_{1n}(\hat{\boldsymbol{\lambda}})$. After some algebra, one can recast (B1) in the following form

$$\begin{aligned}
&\Pi_L^{surf}(\mathbf{q},\mathbf{q};\omega) = \frac{1}{|f_L(q)|^2}\sum_J\sum_{\ell m}\frac{2J+1}{2\ell+1}a^2_{J\ell} \\
&\times\int_0^\infty db\, b\, \mathrm{e}^{-i\chi^*_{\rm opt}(b)}\int_0^\infty db'\, b'\, \mathrm{e}^{i\chi_{\rm opt}(b')}\int_0^\infty d\lambda\,\lambda\, f_L^*(\lambda)\int_0^\infty d\lambda'\,\lambda'\, f_L(\lambda')\Pi_J(\lambda,\lambda';\omega) \\
&\times\mathcal{F}^*_{\ell m}(q;b,\lambda)\mathcal{F}_{\ell m}(q;b',\lambda'),
\end{aligned} \tag{B2}$$

$a_{J\ell}$ being defined in Eq. (2.7a) and

$$\mathcal{F}_{\ell m}(q;b,\lambda) = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\phi_b\int_0^{2\pi} d\phi_\lambda \mathrm{e}^{i(\mathbf{q}-\boldsymbol{\lambda})\cdot\mathbf{b}}Y_{\ell m}(\hat{\boldsymbol{\lambda}}). \tag{B3}$$

$\hat{\boldsymbol{\lambda}}$ is a versor in the (x,y) plane, hence

$$\begin{aligned}
&Y_{\ell m}(\hat{\boldsymbol{\lambda}}) \equiv Y_{\ell m}(\frac{\pi}{2},\phi_\lambda) \\
&= (-1)^m\left[\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}\right]^{\frac{1}{2}}P_\ell^m(0)\mathrm{e}^{im\phi_\lambda} \\
&= \left\{(-1)^m\left(\frac{2\ell+1}{4\pi}\right)^{\frac{1}{2}}\left[\frac{(\ell-m-1)!!(\ell+m-1)!!}{(\ell+m)!!(\ell-m)!!}\right]^{\frac{1}{2}}\right. \\
&\qquad\left.\times\frac{(-1)^{\ell+m}}{2}\left[(-1)^{(\ell+m)/2}+(-1)^{-(\ell+m)/2}\right]\right\}\mathrm{e}^{im\phi_\lambda} \\
&= K_{\ell m}\mathrm{e}^{im\phi_\lambda},
\end{aligned} \tag{B4}$$

where $K_{\ell m}$ is defined as the the quantity in brackets in (B4).

Thus, employing a standard integral representation of the Bessel function, one finds

$$\mathcal{F}_{\ell m}(q; b, \lambda) = (-1)^m K_{\ell m} J_M(qb) J_M(\lambda b) \tag{B5}$$

and

$$\begin{aligned}
&\Pi_L^{surf}(\mathbf{q}, \mathbf{q}; \omega) = \frac{1}{4\pi} \sum_J (2J+1) \\
&\times \int_0^\infty d\lambda\, \lambda\, f_L^*(\lambda) \int_0^\infty d\lambda'\, \lambda'\, f_L(\lambda') \Pi_J(\lambda, \lambda'; \omega) \\
&\times \sum_{\ell m} c_{J\ell m} \int_0^\infty db\, b\, \mathrm{e}^{-i\chi^*_{\rm opt}(b)} J_m(qb) J_m(\lambda b) \int_0^\infty db'\, b'\, \mathrm{e}^{i\chi_{\rm opt}(b')} J_m(qb') J_m(\lambda' b'),
\end{aligned} \tag{B6}$$

with $c_{J\ell m}$ given by Eq. (3.13).

Using the well-known orthogonality relation

$$\int_0^\infty db\, b\, J_m(qb) J_m(\lambda b) = \frac{\delta(q-\lambda)}{\lambda} \tag{B7}$$

one can write

$$\int_0^\infty db\, b\, \mathrm{e}^{i\chi_{\rm opt}(b)} J_m(qb) J_m(\lambda b) = \frac{\delta(q-\lambda)}{\lambda} - g_m(\lambda, q), \tag{B8}$$

having defined $g_m(\lambda, q)$ in Eq. (3.12). Substituting (B8) in (B6), it is then straightforward to obtain Eq. (3.10a).

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FIGURES

FIG. 1. ph self-energy diagrams: (a) single particle diagrams; (b) interference diagrams.

FIG. 2. $\sigma\tau$ longitudinal and transverse response functions of ^{40}Ca at $q = 410$ MeV/c and $q = 330$ MeV/c. Free (dashed line) and RPA (solid line) responses without spreading width and free (dotted line) and RPA (dot-dashed line) responses with spreading width are displayed.

FIG. 3. Structure function of ^{40}Ca at $q = 410$ MeV/c and $q = 330$ MeV/c. Free (dashed line) and RPA (solid line) contributions include the spreading width. Data are from ref. [6].

FIG. 4. $\sigma\tau$ longitudinal response functions of ^{12}C and ^{40}Ca at $q = 1.54$ fm^{-1} and $q = 2.31$ fm^{-1}. Free (dashed line) and RPA (solid line) *volume* responses are reported, together with free (dotted lines) and RPA (dot-dashed lines) *surface* responses for two-values of $\widetilde{\sigma}_{\rm tot}$: $\widetilde{\sigma}_{\rm tot} = 30$ mb (higher curves) and $\widetilde{\sigma}_{\rm tot} = 40$ mb (lower curves). The spreading width is always included. The ^{40}Ca surface responses have been multiplied by a factor 2.

FIG. 5. As in Fig. 4, but for $\sigma\tau$ transverse response functions.

FIG. 6. $\sigma\tau$ surface longitudinal response functions (divided by $N_{\rm eff}$) of ^{12}C and ^{40}Ca at $q = 1.54$ fm^{-1} and $q = 2.31$ fm^{-1} with (SW) and without (no SW) inclusion of spreading width. In each plot the ^{12}C free (dashed line) and RPA (solid line) responses are compared to the ^{40}Ca free (dotted line) and RPA (dot-dashed line) responses.

FIG. 7. Two-step responses of ^{40}Ca with gaussian amplitudes $|f(q)|^2 = A\exp(-\eta q^2)$ at $q = 1.4$ fm^{-1} and $q = 2.4$ fm^{-1} for various values of η: (1) $\eta = 0.001$, (2) $\eta = 0.01$, (3) $\eta = 0.1$, (4) $\eta = 0.5$, (5) $\eta = 1$; the arrows show the position of the QEP. The scale is arbitrary.

FIG. 8. Inelastic (p,n) cross-sections on ^{12}C at q ranging from 1.16 fm^{-1} to 2.31 fm^{-1}. The calculation based on Eq. (4.4) is reported (solid line), together with the separate one-step (dashed line) and two-step (dotted line) contributions and with the calculation based on Eq. (4.5) (dot-dashed line). Data are from the 795 MeV experiment of ref. [1].

FIG. 9. As in Fig. 8, but for inelastic (p,p$'$) cross-sections on ^{12}C at q ranging from 1.42 fm^{-1} to 1.93 fm^{-1}. Data are from the 795 MeV experiment of ref. [2].